VAP2(2009/4/20)

1 Let V be the volume of a cylinder of height h and radius r. If h increases 1% and r decreases 2%, how much percent does V increase or decrease?

Proof. The linear approximation for $V = \pi r^2 h$ is

$$\frac{\Delta V}{V} = 2\frac{\Delta r}{r} + \frac{\Delta h}{h}.$$

If we set $\Delta r/r = -0.02$ and $\Delta h/h = 0.01$, then $\Delta V/V = -0.03$, which means that V decreases 3%. □

2 Consider the temperature distribution in a cubic room given by a function $f(x, y, z) = \sin(x + 2y) \sin z$ $(0 \le x, y, z \le \pi)$. Find the direction of heat flow at the point $(x, y, z) = (\pi/3, \pi/6, \pi/3)$. Hint: Heat flows from a higher temperature region to a lower one.

Proof. The direction of heat flow is given by

$$-\nabla f = (-\cos(x + 2y) \sin z, -2\cos(x + 2y) \sin z, -\sin(x + 2y) \cos z),$$

which is evaluated at the point $(\pi/3, \pi/6, \pi/3)$ to get

$$\begin{aligned}(-\cos(2\pi/3) \sin(\pi/3), -2\cos(2\pi/3) \sin(\pi/3), -\sin(2\pi/3) \cos(\pi/3)) \\ = (\sqrt{3}/4, \sqrt{3}/2, -\sqrt{3}/4) = \frac{\sqrt{3}}{4}(1, 2, -1).\end{aligned}$$

Thus heat flows according to the direction of $(1, 2, -1)$. □