1 For the path $\phi : \mathbb{R} \ni t \mapsto (t, \cos t, \sin t) \in \mathbb{R}^2$, determine whether the tangential line at $t = \pi/2$ passes through the point $(0, \frac{3}{2}, 1)$ or not.

Proof. Since the velocity vector

$$\frac{d}{dt}(t, \cos t, \sin t) = (1, -\sin t, \cos t)$$

takes the value $(1, -1, 0)$ at $t = \pi/2$, the tangential line in question is

$$(x, y, z) = (\pi/2, 0, 1) + t(1, -1, 0), \qquad -\infty < t < \infty.$$

If this contains the point $(0, 3/2, 1)$, we should have t satisfying

$$(\pi/2 + t, -t, 1) = (0, 3/2, 1),$$

which is imposible. Thus the tangential line does not pass through $(0, 3/2, 1)$. □

2 Calculate the arc length of the path $[0, 1] \ni t \mapsto (4t, 4e^t, e^{2t})$ in $\mathbb{R}^3$.

Proof. Since the velocity vector is

$$\frac{d}{dt}(4t, 4e^t, e^{2t}) = (4, 4e^t, 2e^{2t})$$

with its magnitude equal to

$$\sqrt{4^2 + (4e^t)^2 + (2e^{2t})^2} = 2\sqrt{4 + 4e^{2t} + e^{4t}} = 4 + 2e^{2t},$$

the arc length is calculated by

$$\int_0^1 (4 + 2e^{2t})\, dt = 4 + \left[e^{2t}\right]_0^1 = e^2 + 3.$$

□